PERIOD AND FREQUENCY OF S.H.M

Periodic time. A simple harmonic motion is represented by the equation.

$$Y = a \sin(\omega t + \phi)$$

In this equation if we increase t by $\frac{2\pi}{\omega}$, then

$$Y = a \sin\left[\omega\left(t + \frac{2\pi}{\omega}\right) + \phi\right]$$

$$= a \sin(\omega t + 2\pi + \phi]$$

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i.e the displacement of the particle after a time $T = \frac{2\pi}{\omega}$ is the same.

Hence T gives the periodic time of the simple harmonic oscillator.

$$T = \frac{2\pi}{\omega}$$

and frequency $n = \frac{1}{T} = \frac{\omega}{2\pi}$

Thus $\omega = 2\pi n$ = angular velocity of the harmonic oscillator. The acceleration of a simple harmonic oscillator is given by the relation

$$\frac{d^2y}{dt^2} = -\omega^2 y$$

Neglecting the negative sign, we have

$$\omega^2 = \frac{d^2y}{dt^2} / y = \frac{\text{Acceleration}}{\text{Displacement}}$$

$$\omega = \sqrt{\frac{\text{Acceleration}}{\text{Displacement}}}$$

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}$$

$$\boxed{n = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{\text{Acceleration}}{\text{Displacement}}}}$$